MTH 605: Topology I

Homework VI

(Due 06/11)

1. Show that cone CX of a path-connected space X is contractible.
2. Show that the Mobius band deformation retracts onto its core circle, but not to its boundary circle.
3. Find all covering spaces of the Mobius band up to isomorphism. [Hint: See Example 1.42 in page 74 of Hatcher.]
4. Compute the fundamental group of the following spaces.
   (a) The complement of a finite set of points in $\mathbb{R}^n$, for $n \geq 3$.
   (b) The complement of a union of n lines through the origin in $\mathbb{R}^3$.
   (c) The quotient space of S^2 with its north and south poles identified.
   (d) The quotient space obtained from taking two copies of $S^1 \times S^1$ and identifying the circle $S^1 \times \{x_0\}$ in one torus with the corresponding circle in the other torus.
   (e) The complement of a finite set of points in the closed orientable surface S_g of genus g.
5. Suppose that a space Y is obtained from a path-connected space X by attaching n-cells for a fixed $n \geq 3$. Show that the inclusion $X \hookrightarrow Y$ induces an isomorphism of fundamental groups. Use this to show that the complement of a discrete subspace of $\mathbb{R}^n$ is simply connected for $n \geq 3$.
6. The *mapping torus* T_f of a map $f : X \to X$ is the quotient of $X \times I$ obtained by identifying each point $(x, 0)$ with $(f(x), 1)$. In the case $X = S^1 \vee S^1$ with f basepoint-preserving, compute a presentation for $\pi_1(T_f)$ in terms of the induced map $f_* : \pi_1(X) \to \pi_1(X)$. Do the same when $X = S^1 \times S^1$. [Hint: Regard T_f as built from $X \vee S^1$ by attaching cells.]